
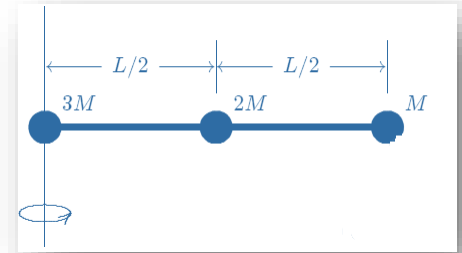
	<b>INDIAN SCHOOL AL WADI AL KABIR</b>	
<b>Class: XI</b>	<b>Department: Science 2022 – 23</b> <b>Subject: Physics</b>	<b>Date of submission:</b> <b>20.11.2022</b>
<b>Worksheet No: 06</b> <b>With Answers</b>	<b>Topic: SYSTEM OF PARTICLES AND</b> <b>ROTATIONAL MOTION</b>	<b>Note:</b> <b>A4 FILE FORMAT</b>
<b>CLASS/SEC.:</b>	<b>NAME OF THE STUDENT:</b>	<b>ROLL NO.:</b>

### OBJECTIVE TYPE QUESTIONS

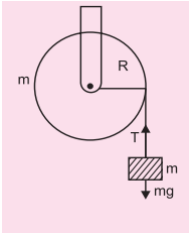
- 1) If the resultant of all external forces is zero, then velocity of centre of mass will be
  - a) Zero
  - b) Constant
  - c) Either (a) or (b)
  - d) Neither (a) or (b)
  
- 2) During summersault, a swimmer bends his body to
  - a) Increase moment of inertia
  - b) Decrease moment of inertia
  - c) Decrease the angular momentum
  - d) Reduce the angular velocity
  
- 3) A boy comes and sits suddenly on a circular rotating table. What will remain conserved for the table – boy system?
  - a) Angular velocity
  - b) Angular momentum
  - c) Linear momentum
  - d) Angular acceleration
  
- 4) Three masses are placed on the axis: 300 g at origin, 500 g at  $x = 40$  cm and 400 g at  $x = 70$  cm. The distance of the centre of mass from the origin is
  - a) 40 cm
  - b) 45 cm
  - c) 50 cm
  - d) 30 cm
  
- 5) One revolution per minute is about:
  - a) 0.0524 rad/s
  - b) 0.105 rad/s
  - c) 0.95 rad/s
  - d) 1.57 rad/s

- 6) A flywheel is initially rotating at 20 rad/s and has a constant angular acceleration. After 9.0 s it has rotated through 450 rad. Its angular acceleration is:
- a) 3.3 rad/s
  - b) 4.4 rad/s
  - c) 5.6 rad/s
  - d) 6.7 rad/s

- 7) Three identical balls, with masses of  $M$ ,  $2M$ , and  $3M$  are fastened to a massless rod of length  $L$  as shown. The rotational inertia about the left end of the rod is:
- a)  $ML^2/2$
  - b)  $ML^2$
  - c)  $3ML^2/2$
  - d)  $3ML^2/4$



- 8) The sum of moments of all the particles in a system about the centre of mass is always
- a) maximum
  - b) minimum
  - c) infinite
  - d) zero
- 9) Which of the following statements are incorrect about centre of mass?
- I.** Centre of mass can coincide with geometrical centre of a body
  - II.** Centre of mass of a system of two particles does not always lie on the line joining the particles
  - III.** Centre of mass should always lie on the body.
- a) II and III
  - b) I and II
  - c) I and III
  - d) I, II and III
- 10) Ten seconds after an electric fan is turned on, the fan rotates at 300 rev/min. Its average angular acceleration is:
- a)  $3.14 \text{ rad/s}^2$
  - b)  $30 \text{ rad/s}^2$
  - c)  $30 \text{ rev/s}^2$
  - d)  $50 \text{ rev/min}^2$
- 11) Three particles of the same mass lie in the  $x$ - $y$  plane. The  $(x, y)$  coordinates of their positions are  $(1, 1)$ ,  $(2, 2)$  and  $(3, 3)$  respectively. The  $(x, y)$  co-ordinates of the centre of mass are :
- a)  $(1, 2)$
  - b)  $(2, 2)$
  - c)  $(4, 2)$
  - d)  $(6, 6)$

- 12) A mass  $m$  is moving with a constant velocity along a line parallel to the X-axis away from the origin, its angular momentum w.r.t. origin :
- is zero
  - is constant
  - goes on decreasing
  - goes on increasing
- 13) A child is standing with folded hands at the center of a platform rotating about its central axis. The kinetic energy of the system is  $K$ . The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is :
- $2K$
  - $K/2$
  - $K/4$
  - $4K$ .
- 14) Which of the following statements is FALSE for a particle moving in a circle with a constant angular speed ?
- The velocity vector is tangent to the circle
  - The acceleration vector is tangent to the circle
  - The acceleration vector points to the centre of the circle
  - The velocity and acceleration vectors are perpendicular to each other.
- 15) Angular momentum of the particle rotating with a central force is constant due to : **(A.I.E.E.E. 2007)**
- Constant linear momentum
  - Zero Torque
  - Constant Torque
  - Constant Force.
- 16) A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass  $m$  and radius  $R$ . If the string does not slip on the cylinder, with what acceleration will the mass fall on release?
- $g$
  - $2/3 g$
  - $g/2$
  - $5/6 g$
- 
- (J.E.E. Main 2014)**
- 17) A bob of mass  $m$  attached to an inextensible string of length  $l$  is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed  $\omega$  rad/s about the vertical. About the point of suspension:
- angular momentum changes in direction but not in magnitude.
  - angular momentum changes both in direction and magnitude.
  - angular momentum is conserved.
  - angular momentum changes in magnitude but not in direction. **(J.E.E. Main 2014)**
- 18) A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc :
- (A.I.E.E.E. Paper-I 2011)**

- a) continuously decreases
  - b) continuously increases
  - c) first increases and then decreases
  - d) remains unchanged
- 19) Two uniform rods of the same diameter, having lengths of 2 m and 3 m, and having linear densities of  $4 \text{ kg m}^{-1}$  and  $6 \text{ kg m}^{-1}$  respectively are joined end to end. The distance of the centre of mass of the combined rod from the centre of mass of the first rod is :
- (a)  $4/3 \text{ m}$
  - (b)  $6/9 \text{ m}$
  - (c)  $45/26 \text{ m}$
  - (d)  $26/45 \text{ m}$
- 20) Four particles of masses 1 kg, 2 kg, 3 kg and 4 kg are at the vertices of a rectangle of sides a and b with  $a > b$ . If  $a = 1 \text{ m}$ ,  $b = 2 \text{ m}$ , what is the location of their centre of mass ?
- (a) 0.5 m, 1.4 m
  - (b) 1.4 m, 0.5 m
  - (c) 0.14 m, 0.05 m
  - (d) 0.05 m, 0.14 m.

**Assertion and Reason:**

- 21) **Statement–I:** The centre of mass of a body may lie at a point where there is no mass.  
**Statement–II:** Centre of mass of a body is a point where whole mass of the body is supposed to be concentrated.
- (a) Statement–I is true, Statement–II is false.
  - (b) Statement–I is false, Statement–II is true.
  - (c) Statement–I is true, Statement–II is true and statement–II is correct explanation of Statement–I
  - (d) Statement–I is true, Statement–II is true and Statement –II is not correct explanation of Statement–I.
- 22) **Statement–I:** The M.I. of a solid sphere about the axis passing through its centre is  $I = \frac{2}{5} mR^2$ , where m is its mass and R is the radius.  
**Statement–II:** M.I. does not depend upon the distribution of mass of the body about axis.
- (a) Statement –I is true, Statement–II is false.
  - (b) Statement– I is false, Statement–II is true.
  - (c) Statement–I is true, Statement–II is true and Statement – II is correct explanation for Statement–I.
  - (d) Statement–I is true, Statement–II is true and Statement–II is not correct explanation of Statement–I.

### **Descriptive type of questions:**

- 23) If no external torque acts on a body, will its angular velocity remain conserved?
- 24) A particle is executing uniform circular motion. Is the particle in rotational equilibrium?
- 25) A particle moves in a circular path with decreasing speed. What happens to its angular momentum?
- 26) Do center of Mars and center of gravity of a body coincide?
- 27) A rope walker holds a long uniform stick in his hands. If he bends to right, in which direction should he turn the stick to keep himself in the balanced position on the rope?

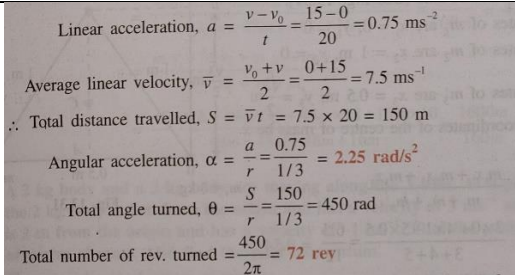
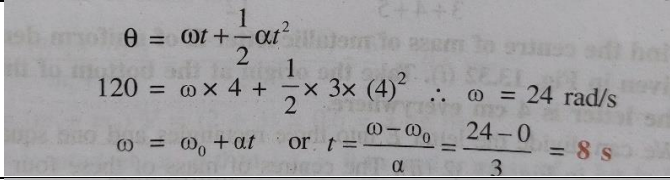
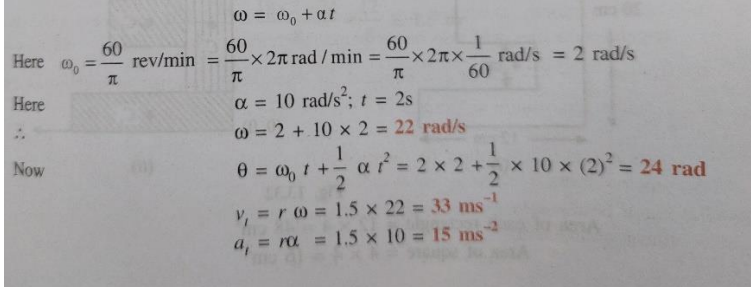
### **Numerical questions:**

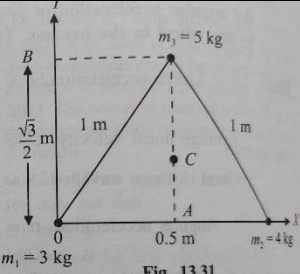
- 28) A car accelerates uniformly from rest to a speed of 15 m/s in a time of 20 s. Find the angular acceleration of one of its wheel and the number of revolutions turned by the wheel in the process. The radius of the car wheel is  $\frac{1}{3}$  meter.
- 29) A wheel starting from rest is rotating with the constant angular acceleration of  $3 \text{ rad/s}^2$ . An observer notes that it traces an angle of 120 radians in four second interval. For how long the wheel had been rotated when the observer started hits observation?
- 30) A wheel of radius 1.5 meter is rotating at a constant angular acceleration of  $10 \text{ rad/s}^2$ . Its initial (at  $t=0$ ) angular speed being  $60/\pi$  revolution per minute. Find its angular speed, angular displacement and linear velocity and acceleration at a point on the rim at time  $t = 2 \text{ s}$ .
- 31) Three particles of masses 3 kg, 4 kg and 5 kg are located at the corners of an equilateral triangle of side 1 meter. Locate the center of mass.
- 32) A 2 kg body and a 3 kg body are moving along the X axis. At a particular instant, the 2 kg body is 1 meter from the origin and has a velocity of 3 m/s and the 3 kg body is 2 meter from the origin, and has the velocity of 1 m/s. Find the position and velocity of center of mass. Also find the total momentum.
- 33) A mass of 4 kg is located at  $x_1 = 0 \text{ m}$ ;  $y_1 = 0 \text{ m}$  and a mass of 6 kg is located at  $x_2=5 \text{ meter}$ ,  $y_2= 4 \text{ m}$ . Where should one place a third mass of 8 kg, so that the coordinates of the center of mass of the system is at  $x = 2 \text{ m}$ ,  $y = 2 \text{ m}$ ?
- 34) A solid flywheel has a moment of inertia of  $0.1 \text{ kgm}^2$  about its axis of rotation. It is set in motion by applying a tangential force of 19.6 N with a rope wound round its circumference. If the radius of the wheel is 10 centimeters, calculate the angular acceleration produced.
- 35) A flying of mass 25KG has a radius of 0.2 meter. It is making 240 rpm. What is the torque necessary to bring it to rest in 20 s? If the torque is due to a force applied tangentially on the rim of the flywheel, what is the magnitude of the force?
- 36) What constant torque should be applied to a disc of mass 16 kg and diameter 0.5 meter, so that it acquires an angular velocity of  $4\pi \text{ rad/s}$  in 8 s? The disc is initially at rest and rotates about an axis through the center of the disc and in a plane, perpendicular to the disc.

- 37) A ballet dancer spins about a vertical axis at 1 rps with arms outstretched. With her arms folded, her moment of inertia about the vertical axis decreases by 60%. Calculate the new rate of revolution.
- 38) If the earth were to suddenly contract to half its present radius, without any external torque acting on it, by how much would the day be decreased?
- 39) A mass of 10 kg connected to the end of a rod of negligible mass is rotating in a circle of radius 30 centimeter, with an angular velocity of 10 rad/s. If this mass is brought to rest in 10 seconds, what is the torque applied?
- 40) Find the angular momentum and rotational kinetic energy of the earth due to its daily rotation about its own axis. Assume earth to be a uniform sphere. Mass of earth  $M = 5.98 \times 10^{24}$  kg and radius of Earth =  $6.37 \times 10^6$  m.

**ANSWERS:**

1	Either (a) or (b)
2	Decrease moment of inertia
3	Angular momentum
4	40 cm
5	0.105 rad/s
6	6.7 rad/s
7	$3ML^2/2$
8	zero
9	II and III
10	$3.14 \text{ rad/s}^2$
11	(2, 2)
12	is constant
13	$K/2$
14	The acceleration vector is tangent to the circle
15	Zero Torque
16	$g/2$
17	angular momentum is conserved
18	first increases and then decreases
19	$45/26$ m
20	0.5 m, 1.4 m
21	Statement–I is true, Statement–II is true and statement–II is correct explanation of Statement–I
22	Statement –I is true, Statement–II is false.
23	We know that $\tau = \frac{dl}{dt}$

	<p>If <math>\tau = 0</math> then <math>\frac{dl}{dt} = 0</math> or <math>l</math> is a constant. Thus it is the angular momentum that is conserved and not the angular velocity of the body</p>
24	<p>It is true. In uniform circular motion the angular velocity of the particle is constant in magnitude and direction. Hence angular acceleration of the particle is 0. We know that a particle is said to be in rotational equilibrium if its angular acceleration is 0.</p>
25	<p>The angular momentum of a particle of mass <math>m</math> moving along a circular path of radius <math>r</math> with the linear velocity <math>v</math> is given by <math>\vec{l} = \vec{r} \times m\vec{v}</math></p> <p>If the speed of the particle decreases along the circular path, the magnitude of angular momentum decreases. However, the direction of angular momentum remains unchanged.</p>
26	<p>They coincide only in uniform gravitational field and not in non-uniform gravitational field.</p>
27	<p>When the walker leans towards to right, he produces a clockwise torque. To keep himself in the balance, he should turn the stick in such a way to produce anticlockwise torque. To do so, he should turn the left end of the stick downward and right end upward.</p>
28	 <p>Linear acceleration, <math>a = \frac{v-v_0}{t} = \frac{15-0}{20} = 0.75 \text{ ms}^{-2}</math></p> <p>Average linear velocity, <math>\bar{v} = \frac{v_0+v}{2} = \frac{0+15}{2} = 7.5 \text{ ms}^{-1}</math></p> <p><math>\therefore</math> Total distance travelled, <math>S = \bar{v}t = 7.5 \times 20 = 150 \text{ m}</math></p> <p>Angular acceleration, <math>\alpha = \frac{a}{r} = \frac{0.75}{1/3} = 2.25 \text{ rad/s}^2</math></p> <p>Total angle turned, <math>\theta = \frac{S}{r} = \frac{150}{1/3} = 450 \text{ rad}</math></p> <p>Total number of rev. turned <math>= \frac{450}{2\pi} = 72 \text{ rev}</math></p>
29	 <p><math>\theta = \omega t + \frac{1}{2}\alpha t^2</math></p> <p><math>120 = \omega \times 4 + \frac{1}{2} \times 3 \times (4)^2 \quad \therefore \omega = 24 \text{ rad/s}</math></p> <p><math>\omega = \omega_0 + \alpha t \quad \text{or} \quad t = \frac{\omega - \omega_0}{\alpha} = \frac{24 - 0}{3} = 8 \text{ s}</math></p>
30	 <p><math>\omega = \omega_0 + \alpha t</math></p> <p>Here <math>\omega_0 = \frac{60}{\pi} \text{ rev/min} = \frac{60}{\pi} \times 2\pi \text{ rad/min} = \frac{60}{\pi} \times 2\pi \times \frac{1}{60} \text{ rad/s} = 2 \text{ rad/s}</math></p> <p>Here <math>\alpha = 10 \text{ rad/s}^2</math>; <math>t = 2\text{s}</math></p> <p><math>\therefore \omega = 2 + 10 \times 2 = 22 \text{ rad/s}</math></p> <p>Now <math>\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 2 \times 2 + \frac{1}{2} \times 10 \times (2)^2 = 24 \text{ rad}</math></p> <p><math>v_t = r \omega = 1.5 \times 22 = 33 \text{ ms}^{-1}</math></p> <p><math>a_t = r\alpha = 1.5 \times 10 = 15 \text{ ms}^{-2}</math></p>

31	<p>equilateral triangle</p> <p>Hint. We choose our coordinate system as shown (to simplify calculations) with <math>m_1</math> at the origin and <math>m_2</math> on the X-axis [See Fig. 13.31].  Coordinates of <math>m_1</math> are <math>x_1 = 0, y_1 = 0</math>  Coordinates of <math>m_2</math> are <math>x_2 = 1 \text{ m}, y_2 = 0</math>  Coordinates of <math>m_3</math> are <math>x_3 = 0.5 \text{ m}, y_3 = \frac{\sqrt{3}}{2} \text{ m}</math>  Let the coordinates of the centre of mass be <math>x</math> and <math>y</math>.</p>  <p><math>\therefore x = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}</math></p> $= \frac{3 \times 0 + 4 \times 1 + 5 \times 0.5}{3 + 4 + 5} = \frac{6.5}{12} = 0.54 \text{ m}$ <p>and <math>y = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}</math></p> $= \frac{3 \times 0 + 4 \times 0 + 5 \times \frac{\sqrt{3}}{2}}{3 + 4 + 5} = \frac{5 \times \sqrt{3}}{12} = 0.36 \text{ m}$ <p style="text-align: right;">Fig. 13.31</p>
32	<p>Hint. The <math>x</math>-coordinate of centre of mass is</p> $x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{2 \times 1 + 3 \times 2}{2 + 3} = \frac{8}{5} = 1.6 \text{ m}$ <p>The velocity of the centre of mass is</p> $V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{2 \times 3 + 3 \times (-1)}{2 + 3} = \frac{3}{5} = 0.6 \text{ ms}^{-1}$ <p>Total momentum <math>= (m_1 + m_2) V = (2 + 3) \times 0.6 = 3 \text{ kg ms}^{-1}</math></p>
33	<p>Hint. <math>m_1 = 4 \text{ kg}</math> ; <math>m_2 = 6 \text{ kg}</math> ; <math>m_3 = 8 \text{ kg}</math></p> $\therefore x = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{4 \times 0 + 6 \times 5 + 8 \times x_3}{4 + 6 + 8} = \frac{30 + 8x_3}{18}$ <p>or <math>x_3 = \frac{18x - 30}{8} = \frac{18 \times 2 - 30}{8} = \frac{6}{8} = 0.75 \text{ m}</math></p> <p>Now <math>y = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{4 \times 0 + 6 \times 4 + 8 \times y_3}{4 + 6 + 8} = \frac{24 + 8y_3}{18}</math></p> $\therefore y_3 = \frac{18y - 24}{8} = \frac{18 \times 2 - 24}{8} = \frac{12}{8} = 1.5 \text{ m}$
34	<p><b>Solution.</b> <math>I = 0.1 \text{ kg m}^2</math> ; <math>F = 19.6 \text{ N}</math> ; <math>R = 10 \text{ cm} = 0.1 \text{ m}</math>  Applied torque, <math>\tau = F \times R = 19.6 \times 0.1 = 1.96 \text{ Nm}</math></p> <p>Now, <math>\tau = I\alpha</math></p> $\therefore \text{Angular acceleration, } \alpha = \frac{\tau}{I} = \frac{1.96}{0.1} = 19.6 \text{ rad s}^{-2}$
35	<p><b>Solution.</b> Mass of flywheel, <math>M = 25 \text{ kg}</math> ; Radius of flywheel, <math>R = 0.2 \text{ m}</math>  Moment of inertia of the flywheel about the axis through its centre and perpendicular to its plane is</p> $I = \frac{1}{2} MR^2 = \frac{1}{2} (25) \times (0.2)^2 = 0.5 \text{ kg m}^2$ <p>Initial angular velocity, <math>\omega_0 = 2\pi f_0 = 2\pi \times \frac{240}{60} = 8\pi \text{ rad s}^{-1}</math>  Final angular velocity, <math>\omega = 0</math></p> <p>Now <math>\omega = \omega_0 + \alpha t</math></p> $\therefore \alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 8\pi}{20} = -0.4\pi \text{ rad s}^{-2}$ <p>Torque on the flywheel, <math>\tau = I\alpha = (0.5) \times (-0.4\pi) = -0.2\pi \text{ Nm}</math></p> $\tau = FR$ $\therefore \text{Tangential force, } F = \frac{\tau}{R} = \frac{0.2\pi}{0.2} = \pi \text{ N}$



36	<p><b>Solution.</b> <math>M = 16 \text{ kg}</math>; <math>R = \frac{0.5}{2} = 0.25 \text{ m}</math>; <math>\omega = 4\pi \text{ rad s}^{-1}</math>; <math>t = 8 \text{ s}</math>; <math>\omega_0 = 0</math></p> <p>M.I. of the disc about an axis through its centre and perpendicular to its</p> $I = \frac{1}{2}MR^2 = \frac{1}{2} \times (16) \times (0.25)^2 = 0.5 \text{ kg m}^2$ <p>Angular acceleration, <math>\alpha = \frac{\omega - \omega_0}{t} = \frac{4\pi - 0}{8} = \frac{\pi}{2} \text{ rad s}^{-1}</math></p> <p>Now, <math>\tau = I\alpha = (0.5) \times \left(\frac{\pi}{2}\right) = \frac{\pi}{4} \text{ Nm}</math></p>
37	<p><b>Soluton.</b> Initial angular velocity, <math>\omega_1 = 1 \text{ r.p.s.}</math></p> <p>Initial M.I. of dancer, <math>I_1 = I</math> (say)</p> <p>Final M.I. of dancer, <math>I_2 = 0.4 I</math></p> <p>Let <math>\omega_2</math> r.p.s. be the new angular velocity. Since angular momentum remains constant,</p> $\therefore I_2\omega_2 = I_1\omega_1$ <p>or <math>\omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left(\frac{I}{0.4I}\right) \times 1 = 2.5 \text{ r.p.s.}</math></p>
38	<p><b>Solution.</b> During contraction, the angular momentum of the isolated earth is conserved.</p> $\therefore I_1\omega_1 = I_2\omega_2$ <p>or <math>\frac{2}{5}MR_1^2\omega_1 = \frac{2}{5}MR_2^2\omega_2</math> or <math>R_1^2\omega_1 = R_2^2\omega_2</math></p> <p>Here <math>R_2 = \frac{R_1}{2}</math>; <math>\omega_1 = \frac{2\pi}{24}</math>; <math>\omega_2 = \frac{2\pi}{T}</math></p> $\therefore R_1^2 \times \frac{2\pi}{24} = \left(\frac{R_1}{2}\right)^2 \times \frac{2\pi}{T} \text{ or } T = 6 \text{ hours}$ <p>This means the day will be of 6 hours duration. Hence the day will decrease by <math>24 - 6 = 18 \text{ hours.}</math></p>
39	<p>M.I. of the mass about the symmetry axis of rotation is</p> $I = MR^2 = 10 \times (0.3)^2 = 0.9 \text{ kg m}^2$ <p>Angular retardation, <math>\alpha = \frac{\Delta\omega}{\Delta t} = \frac{10}{10} = 1 \text{ rad/s}^2</math></p> <p><math>\therefore</math> Applied torque, <math>\tau = I\alpha = 0.9 \times 1 = 0.9 \text{ Nm}</math></p>
40	<p>For earth we have moment of inertia as</p> $I = \frac{2}{5}mR^2$ <p>and angular velocity as</p> $\omega = \frac{2\pi}{T}$ <p>Thus angular momentum is given as</p> $L = \frac{2}{5}mR^2 \times \frac{2\pi}{T}$ <p>Substituting the values of the parameters</p> $L = \frac{2}{5}(5.978 \times 10^{24}\text{kg})(6.378 \times 10^6\text{m})^2 \times \frac{2\pi}{86400\text{s}}$ <p>or</p> $L = 7 \times 10^{33}\text{kg/m}^2/\text{sec}$

Earth completes one rotation about its axis in 1 day.

$$\therefore \omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60} \text{ rad/s}$$

$$E = \frac{1}{2} I \omega^2 = \frac{1}{2} \times \frac{2}{5} M R^2 \omega^2$$

$$= \frac{1}{5} \times \frac{6 \times 10^{24} (6.4 \times 10^6)^2 \times (2\pi)^2}{(24 \times 60 \times 60)}$$

$$= 2.6 \times 10^{29} \text{ J}$$

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